B.TECH. (BIO TECH. ENGINEERING)- ASSIGNMENT

Course Code: Math104C **Course title:** Mathematics-II **Semester:** II[nd]

UNIT-I

1. Discuss the continuity of function f(x) where f(x) is defined by

(i) $f(x) = x - 5$ **(ii)** $f(x) = \frac{x^2-25}{x+5}, x \neq 5$

(iii) $f(x) = \begin{cases} 3 & if\ 0 \le x \le 1 \\ 4 & if\ 1 < x < 3 \\ 5 & if\ 3 \le x \le 10 \end{cases}$ **(iv)** $f(x) = \begin{cases} 2x & \text{if } x < 0 \\ 0 & \text{if } 0 \le x \le 1 \\ 4x & \text{if } x > 1 \end{cases}$

2. Find the discontinuity of function f(x), where f(x) is defined by

(i)$f(x) = \begin{cases} 2x + 3 & if\ x \le 2 \\ 2x - 3 & if\ x > 2 \end{cases}$ (ii) f(x)=$\begin{cases} \frac{x}{|x|} & if\ x < 0 \\ -1 & if\ x \ge 0 \end{cases}$

3. Differentiate the functions with respect to x

(i) $\sin(x^2 + 5)$ (ii) $\frac{\sin(ax+b)}{\cos(cx+d)}$ (iii) $\cos x^3 \sin^2 x^5$ (iv) $2(\cot x^2)^{1/2}$ (v) $\cot(x)^{1/2}$

3 Find $\frac{dy}{dx}$ of the following

(i) $y = \tan^{-1}\frac{3x-x^3}{1-3x^2};\ -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$ (ii) $y = \cos^{-1}\frac{1-x^2}{1+x^2};\ 0 < x < 1$

(iii) $\sin^2 y + \cos xy = k$ (iv) $\sin^2 x + \cos^2 y = 1$ (v) $y = \sin^{-1}(2x\sqrt{i - x^2})$

4 Differentiate with respect to x

(i) $e^{\sin^{-1}x}$ (ii) e^{x^3} (iii) $\sin(\tan^{-1}x\, e^{-x})$ (iv) $\log(\cos e^x)$

5 Differentiate the functions

(i) $\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$ (ii) $(\log x)^{\cos x}$ (iii) $x^x - 2^{\sin x}$ (iv) $(x + \frac{1}{x})^x + x^{(x+\frac{1}{x})}$

6 If u, v, w are functions of x, then show that

$$\frac{d}{dx}(u, v, w) = \frac{du}{dx}\, v.w + u.\frac{dv}{dx}.w + u.v.\frac{dw}{dx}$$

In two ways – first by repeated application of product rule, second by logarithmic differentiation.

UNIT-II

1. Verify Rolle's theorem

(i) $f(x) = x^2 + 2x - 8;\ x \in [-4,2]$ (ii) $f(x) = x^2 + 2;\ x \in [-2,2]$

2. Verify Mean value theorem for the functions

(i) $f(x) = x^2;\ x \in [2,4]$ (ii) $f(x) = x^2 - 4x - 3;\ x \in [1,4]$

3. A ladder 5m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall ate the rate of 2m/s. How fast its height on the wall decreasing when foot of the ladder is 4m away from the wall.
4. The total revenue in rupees received from the sale of x-units of an item is given by
$$R(x) = 13x^2 + 26x + 15$$
Find the marginal revenue when x=7.
5. Find the intervals in which the following functions are strictly decreasing or increasing:
(i) $x^2 + 2x + 5$ (ii) $10 - 6x - 2x^2$ (iii) $-x^3 - 9x^2 - 12x + 1$ (iv) 6-9x$-x^2$
6. Show that y=log(1+x)-$\frac{2x}{2+x}$, x>-1 is an increasing function of x throughoxuxt its domain.
7. Find the slope of the tangent to the curve y=$x^3 - 3x + 2$ at the point whose x-co-ordinate is 3.
8. Find the point on the curve $\frac{x^2}{9} + \frac{y^2}{16} = 1$ at the tangents are
(i) parallel to x-axis (ii) parallel to y-axis
9. Find the maximum and minimum values, if any, of the following function given by
(i) $f(x) = |x + 2| - 5$ (ii) g (x)= -|x+1|+3 (iii) h(x)= sin(2x)+5
(iv) f(x)= |sin(4x+3)| (v) h(x)= x+1;$x \in (-1,1)$

UNIT-III

1. Find the following integrals

(i) $\int 4(e^{3x} + 1)dx$ (ii) $\int (\sqrt{x} - \frac{1}{\sqrt{x}})^2 dx$ (iii) $\int \frac{2-3sinx}{cos^2 x} dx$
(iv) $\int x\sqrt{(x+2)}\ dx$ (v) $\int \frac{1}{x(logx)^m} dx$ (vi) $\int \frac{x^2}{(2+3x^3)^3} dx$

2. Integrate the functions

(i) $f(x) = \frac{x^2}{(x^6+a^6)^{1/2}}$ (ii) $f(x) = \frac{1}{\sqrt{(x^2+2x+2)}}$ (iii) $f(x) = \frac{1}{(x-a)^{1/2}(x-b)^{1/2}}$
(iv) $f(x) = \frac{5x-2}{1+2x+3x^2}$ $(v)\ f(x) = \frac{5x}{(x+1)(x^2-4)}$ (vi) $f(x) = \frac{3x-1}{(x+2)^2}$
(vii) $f(x) = \frac{1}{x(x^n+1)}$ (viii) $f(x) = x(log\ x)^2$ (ix) $f(x) = \frac{xcos^{-1}x}{(1-x^2)^{1/2}}$

UNIT-IV

1 find the degree and order of the differential equations

(i)$\frac{d^4y}{dx^4} + sin(y''') = 0$ (ii) $\frac{d^2y}{dx^2} = cos\ 3x + sin\ 3x$

(iii)$(y''')^2 + (y'')^3 + (y')^4 + y^5$

2 Verify that the given functions is a solution of corresponding equations

$(i)\ \ y = e^x + 1; \qquad y'' - y' = 0 \qquad (ii)\ y = x^2 + 2x + c; \quad y' - 2x - 2 = 0$

(iii)$xy = log\ y + c;\ \ y' = \frac{y^2}{1-xy}(xy \neq 1)$

3 From the differential equation of the family of parabolas having vertex at origin and

axis along positive y-axis.

4 Find the differential equation of the family of circles having centre on y-axis and radius 3 unit.

5 For each differential equation, find general solution

(i)$\frac{dy}{dx} + y = 1$ (ii) $x^5\frac{dy}{dx} = -y^5$ (iii) $\frac{dy}{dx} = (1 + x^2)(1 + y^2)$

6 Solve cos $(\frac{dy}{dx})$ = a; y=1 when x=0

7 The volume of the spherical balloons being inflated changes at a constant rate. If initially radius is 3 unit and after 3 second it is 6 unit. Find the radius of balloons after t seconds.

8 Show that the differential equations is homogenous and solve each of them

(i) $(x^2 + xy)dy = (x^2 + y^2)dx$

(ii) $\left\{x\ cos\ (\frac{y}{x}) + y\ sin\ (\frac{y}{x})\right\}ydx = \left\{-x\ cos\ (\frac{y}{x}) + y\ sin\ (\frac{y}{x})\right\}xdy$

9 Solve $\frac{dy}{dx} - \frac{y}{x} + cosec\ (\frac{y}{x}) = 0;\ y = 0\ when\ x = 1$

10 Solve $2xy + y^2 - 2x^2\frac{dy}{dx} = 0;\ \ y = 2, when\ x = 1$